Szeged WORkshop on Discrete Structures

SZTE JGYPK
University of Szeged
Juhász Gyula Faculty of Education

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UNIVERSITY OF SZEGED
INSTITUTE OF INFORMATICS

SWORDS 2023

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University of Szeged

PREFACE

Dear Participants,

Following the organization of SWORDS 2017, we had prepared everything for the next event, which was initially scheduled for 2020. Unfortunately, the pandemic foiled our plans and we had to cancel that workshop. However, we remained committed to our plans, and started organizing a new event at the beginning of this year. As a result of our efforts, we are happy to announce the

Szeged WORkshop on Discrete Structures (SWORDS 2023)
to be held on
23 - 24 November 2023.

In our call for papers, we opened the “scope-door” wide, and true to the name of our workshop, we accepted talks from the fields of discrete models and methods represented by significant results in graph theory, artificial intelligence, data mining, and other areas of combinatorial optimization. Similarly, our aim was to accept papers both from theory and application. Naturally, since we are in Szeged, you will find results on exact and approximation algorithms among the talks as well.

We are exceptionally proud this year to host researchers as invited speakers at SWORDS 2023 who are among the best in their research area. Concerning the participants, we have two non-independent objectives: we have invited professionals who are leading experts in their research area, and we intend to provide an opportunity to the “new generation” as well; PhD students with relevant results also have the opportunity to present in the sections. We hope that our two-day workshop provides an opportunity to “warm up” professional and social connections among researchers coming from 28 institutions of 12 countries, and result in new joint research and publications in the future.

The financial foundation of our workshop is stable, largely thanks to our sponsors. The National Research, Development and Innovation Office (NKFIH) provided a very generous financial sponsorship for organizing a successful event through the NKFIH Fund No. SNN-135643. Similarly, we are grateful to the Faculty of Education of the University of Szeged for the organizational background, and of course, for hosting the welcome reception. The Public Foundation for the Community of the City of Szeged also sponsored our workshop, and this way, in addition to the University of Szeged, we also have the city among our supporters.

We would like to thank everyone who contributed to the organisation of the workshop, including the providers of the catering services, the hotels accommodating our continuously changing demands with great flexibility, and, most importantly, we are grateful to all participants for coming to Szeged.

We welcome you to the workshop, and wish you a pleasant stay in Szeged.

Organizers of the SWORDS

WELCOME

Ladies and Gentlemen, Distinguished Guests, Dear Colleagues,

On behalf of the Juhász Gyula Faculty of Education at the University of Szeged I SWORD You, that is, **S**incerely **W**elcome **Y**ou **O**n this **R**egular **D**iscussion of **S**pecialists. The abbreviation SWORDS stands for the Szeged WORkshop on Discrete Structures, however it also has a symbolic meaning:

- *Swords are like pieces of history you can hold in your hand.* This workshop has a history going back to 2011 and the workshop is held at the Juhász Gyula Faculty of Education which is dating back to 1873, celebrating its 150th anniversary this year. Within these walls you can experience this proud traditional academic environment with a rich past. At the same time a modern, open and innovative community awaits you, where the professors are student-centred, use practice-oriented innovative pedagogy and the academic courses offer competitive, research-based knowledge and skills. However, I seem to talk like a proud parent, the latest official news are about our Faculty being among the 10 most popular rural faculties in Hungary, and the University of Szeged, within which our Faculty provides the lion's share of training educators, meaning kindergarten educators, lower primary teachers, special needs education teachers and different subject teachers, is the best educator training institute in Hungary, and in the 501-600 range in the world according to the Times Higher Education ranking.
- *Swords are sharp* just like this workshop's content, and your messages on discrete optimization, graph theory, artificial intelligence, data mining, other fields of combinatorial optimization and exact and approximation algorithms can cut through the noise and capture the audience's attention.
- *Just as a skilled swordsman manoeuvres with precision and can leave a lasting impact,* the organisers invited speakers who are among the best of their research topic, either as already leading experts of their research area, or the representatives of the new generation of leading experts to be, who will inspire, persuade, or inform the audience, making a memorable impression.

- *A sword can be versatile, serving various purposes in different situations.* Accordingly, we hope that our two-days' workshop gives the opportunity to socialise and build a research network of researchers from 28 institutions and 12 countries.
- *Just as a sword should be handled with care,* this workshop has sponsors, who helped making this sword even more shiny: The Alfréd Rényi Institute of Mathematics offered professional as well as financial sponsorship, the Public Foundation for the Community of the City of Szeged also sponsored the workshop, and the Juhász Gyula Faculty of Education at the University of Szeged offers the organizational background, and, as the most important part of every conference, the welcome reception.

I extend my heartfelt gratitude to all of you for being a part of this workshop and thank the Department of Applied Informatics at the Institute of Vocational Training, Adult Education and Knowledge Management for organising and hosting this valuable event.

Once again, welcome to the Szeged Workshop on Discrete Structures. May your time together be filled with stimulating discussions, valuable insights, and the forging of lasting connections.

Dr. Klára Tarkó
Vice Dean
Juhász Gyula Faculty of Education
University of Szeged

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ABSTRACTS

Auctions and Bidding

Bo Chen[a]

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Auction theory provides an explicit model of price making and auctions are of considerable practical significance. Auction theory is closely linked to game theory, combinatorial optimization and computational complexity. In the first half of my talk, I will give a brief overview of auction theory and practice. I will introduce some key concepts and results in auction theory, then provide some examples of best auction practice, and conclude with pointers to some seminal full-review articles. In the second half of my talk, I will present my recent studies on some auction problems in the electricity capacity market.

Counting Steiner triple systems

Daniel Heinlein[a], Patric R. J. Östergård[a]

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Terry Griggs observed in his survey paper [1] that the enumeration of Steiner triple systems (STSs) proceeded by centuries: the two STS(13)s were known in the 19th century, the 80 STS(15)s in the 20th, and the STS(19)s in the 21st. He further speculated that the number of STS(21)s might have to wait until the 22nd century or the general availability of quantum computing. This is not the case; the STS(21)s have now been counted with traditional computers (in 82 core-years). Computational approaches for counting STSs will here be discussed. These lead to an algorithm that has been used to obtain the number of isomorphism classes of STS(21)s, 14,796,207,517,873,771. The full article with this result has now been published [2].

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Machine Learning Techniques for Solving Combinatorial Optimisation Problems

Deepak Ajwani[a], Paula Carroll[a], James Fitzpatrick[a],
Saurabh Ray[b], Dena Tayebi[a]

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In recent years, machine learning techniques are being increasingly used for solving combinatorial optimisation problems. This often requires a deep integration between techniques from optimisation literature, algorithm engineering and machine learning. For instance, while the optimisation and algorithmic literature guides the feature engineering in learning models, the learning models can guide crucial design steps in exact MILP solvers as well as heuristics.

Specifically, I would like to talk about the research done in my group on a range of fundamental combinatorial optimisation problems such as set cover, k-median, facility location, vehicle routing problems, Max Cut, Max Clique, Steiner tree etc. Firstly, I will describe a simple supervised learning framework called learning-to-prune that can be used to reduce the size of the problem instances. This enables the computation of high quality solutions to larger and harder instances of combinatorial optimisation problems. Later, I will present a heuristic based on reinforcement learning that provides close-to-optimal solutions on many NP-hard vehicle routing problem variants.

If time permits, I can also point out some opportunities in using machine learning for discovering combinatorial structures.

Exact Methods for the Longest Induced Cycle Problem

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Our primary focus in this work is dedicated to addressing the challenge of identifying the longest induced cycle problem which is classified as NP-complete problem. For a graph $G = (V, E)$ and a subset $W \subseteq V$, the W-induced graph $G[W]$ comprises all the vertices from set W and the edges from G that connect vertices exclusively within W. The objective of the longest induced cycle problem is to determine the largest possible subset W for which the graph $G[W]$ forms a cycle.

Our work proposes three mixed-integer linear programs designed to handle the longest induced cycle problem within general graphs. Some of these models build upon the models applied by prior work focused on solving the longest induced path problem, as seen in the work of Marzo et al.[2] and Bokler et al. [3].

To demonstrate and evaluate the effectiveness of the proposed methods, we present numerical results for the three models. Furthermore, we conducted a comparison between our models and models from [1] to highlight the efficiency of our approach in comparison to existing methods.

References

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On the core of many-to-one assignment games

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Many-to-one assignment games are models of two-sided matching markets where agents on one side are allowed to cooperate with more than one agent from the other side up to a given capacity. For example, in a job market situation, firms want to hire many workers, up to each firm's capacity, but each worker can work for only one firm. We assume that firms value groups of workers additively, so the main data of the market is the value that each firm-worker pair can attain when matched. This value can be freely transferred among the agents. The core is the set of those payoff allocations of the maximum total profit attainable in the market which are stable against deviations by any group of agents.

Although it is well-known that the core of this model is non-empty, the structure of the core has not been fully investigated. To the known dissimilarities with the one-to-one assignment game, we add that the bargaining set does not coincide with the core and the kernel may not be included in the core. Besides, not all extreme core allocations can be obtained by means of a lexicographic maximization or a lexicographic minimization procedure, as it is the case in the one-to-one assignment game.

On the positive side, the maximum and minimum core allocations are characterized by means of the longest length paths and the shortest length paths in a given directed graph. Regarding the remaining extreme core allocations of the many-to-one assignment game, we propose a lexicographic procedure that, for each order on the set of workers, sequentially maximizes or minimizes each player's core payoff. This procedure provides all extreme core allocations.

Robust coloring and related invariants of graphs

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A 1-selection f of a graph G is a function $f : V(G) \to E(G)$ such that $f(v)$ is incident to v for every vertex v. The 1-removed graph G_f has vertex set $V(G)$ and edge set $E(G) \setminus f(V(G))$. Motivated by applicability in extremal graph theory, we define the (1-)robust chromatic number $\chi_1(G)$ as the minimum of $\chi(G_f)$ over all 1-selections f of G. There is a natural analogous way to introduce the robust versions of many other graph invariants, too. We initiate a systematic study of this new area. Among various results, we compare the new parameters with the traditional ones, obtaining estimates that are tight in many cases. The definitions may also be extended to s-valued $V(G) \to E(G)$ mappings called s-selections, for any $s > 1$.

Deep Learning-Based Approximation of Optimal Traveling Salesman Tour Length

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This study introduces a deep learning approach to solve the Traveling Salesman Problem (TSP) in non-Euclidean spaces. Our main objective is to train a deep neural network to efficiently estimate the route length by understanding the problem's topology. We use a dataset of 14.4 million TSP instances obtained from real-world problems, previously solved using brute-force methods, covering problem sizes from 4 to 12 cities. To address this problem, we employed a five-layer multilayer perceptron with the ReLU activation function, trained for 4096 epochs. Our input data comprises a matrix representing the distances between cities. Depending on the problem size, our model achieved an average deviation from optimal solutions ranging from 2% to 2.9%. Additionally, the R^2 metric between predicted and target values is 0.992.

While this initial experiment shows promising results, it's just the first step toward estimating solutions for more complex vehicle routing problems.

Improving container handling in port operations

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This paper examines a method aimed at improving the handling of containers in ports. We present a truck scheduling mathematical model designed to operate within the constraints and limitations typical of port environments. The model is structured to handle concurrent tasks and is tested through simulations to evaluate its performance in realistic settings.

Acknowledgements: This work is supported by the Slovenian Research Agency, research program P1-0404.

Balázs Dávid and Miklós Krész gratefully acknowledge the support of the Slovenian Research and Innovation Agency (ARIS) through the grant N1-0223. They have been also supported by the research program CogniCom (0013103) at the University of Primorska.

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New Lower Bounds for Certain Relaxed Online Packing Problems

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Packing problems are optimization problems that involve finding efficient ways to arrange items into containers. In the case of online problems the items arrive one by one and the decisions are irrevocable. We prove new lower bounds for two online packing problems that have some flexibility [1]. In the online removable multiple knapsack problem, we have a fixed number of identical bins and a sequence of items with sizes and values that arrive online. The goal is to maximize the total value of the items that are packed in the bins. An online algorithm can reject an item without packing it, or remove a previously packed item at any time. In the online minimum peak appointment scheduling an online algorithm must assign a position to each item upon its arrival, where the position is an interval of length equal to the item's size within a bin; minimizing the maximum number of overlapping intervals at any point in time.

Acknowledgements: The research of J. Balogh has been supported by the grant TKP2021-NVA-09 of the Ministry for Innovation and Technology, Hungary.

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Constructing bounded degree graphs with prescribed degree and neighbor degree sequences

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Let $D = \{d_i\}$ and $F = \{f_i\}$ be two sequences of positive integers. We consider the following decision problems: is there a *i*) multigraph, *ii*) loopless multigraph, *iii*) simple graph, *iv*) cycle-free graph (forest or tree), *v*) caterpillar $G = (V, E)$ such that for all k, $d(v_k) = d_k$ and $\sum_{w \in \mathcal{N}(v_k)} d(w) = f_k$ ($d(v)$ is the degree of v and $\mathcal{N}(v)$ is the set of neighbors of v). Here we show that all these decision problems can be solved in polynomial time if $\Delta := \max_k d_k$ is bounded. The problems are converted into an integer programming feasibility problem in which both the number of variables and the number of inequalities depend only on Δ but not on n. The problem is motivated by NMR spectroscopy of hydrocarbons. The algorithm has been implemented in the ZIMPL language, and its applicability is demonstrated on trees up to $n = 1000$ vertices. The average reconstruction time for trees with 1000 vertices is still less than 40 milliseconds.

The talk is based on the publication by the same authors, *Discrete Applied Mathematics, 2023;332:47–61.* .

Acknowledgements: I.M. was supported by NKFIH grants KH126853, K132696 and SNN135643. The project is a continuation of the work done at the 2020 Budapest Semesters in Mathematics. All authors would like to thank the BSM for running the program.

On the starting point of the constraint generation algorithm for submodular function maximization

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Submodular function maximization (SFM) is a central problem in combinatorial optimization and in many cases it involves graphs on which the maximization is defined. It is a well-studied problem, so there are many proposed algorithms in the literature. The greedy strategy quickly finds a feasible solution guaranteeing a $(1 - 1/e)$ approximation. However, there are many applications that expect an optimal result within a reasonable computational time. One well-known method for finding the global optimum is the constraint generation (CG) algorithm [1]. The CG works on the MIP formulation of the problem which might have exponential many constraints. Traditionally, the initial feasible solution of the CG is given by the greedy algorithm.

In our recent work [2], we created different versions of the CG algorithm using some heuristic steps which are derived from the graph structure or the definition of SFM. The focus of our current research is the starting point of the CG algorithm. It turns out that choosing different starting point than the greedy solution might give more efficient solution in terms of running time. Advantages of using the previous heuristics rules will be demonstrated.

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Shannon capacity, Lovász theta number and the Mycielski construction

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The Shannon OR-capacity $C_{\mathrm{OR}}(G)$ of a graph G is defined as $C_{\mathrm{OR}}(G) = \lim_{t\to\infty} \sqrt[t]{\omega(G^t)}$ where G^t is an appropriately defined graph exponentiation (and ω stands for clique number). In [1] Lovász proved $C_{\mathrm{OR}}(C_5) = \sqrt{5}$ with the help of his famous ϑ-number introduced in [1]. The Mycielski construction is one of the standard constructions showing that a triangle-free graph can have arbitrarily large chromatic number. From a graph G it produces graph $M(G)$ having the same clique number while the chromatic number increases by 1. We investigate the effect of this construction on the complementary Lovász theta number $\overline{\vartheta}(G) = \vartheta(\overline{G})$ and on Shannon OR-capacity. For the former we prove that $\overline{\vartheta}(M(G))$ is determined by $\overline{\vartheta}(G)$ and give an explicit formula for it in terms of $\overline{\vartheta}(G)$. For Shannon OR-capacity we show that $C_{\mathrm{OR}}(M(G)) > C_{\mathrm{OR}}(G)$ whenever there exists a $k \in \mathbb{N}$ such that $C_{\mathrm{OR}}(G) = \sqrt[k]{\omega(G^k)}$.

The talk is based on the forthcoming paper [2].

Acknowledgements: We thank Anna Gujgiczer for useful discussions.

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Board Packing

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We define a rectangle packing model which is a common generalization of several other packing models.

We show that it is NP-hard, and discuss its different properties.

To solve it, we write up its model and apply Cplex, for relatively small instances.

We also build a Genetic Algorithm (GA) which can be efficient also when Cplex fails.

Many experimental results are provided and other possible options are discussed that can be applied for solution.

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Finding Triangles or Independent Sets

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We revisit the algorithmic problem of finding a triangle in a graph <Triangle Detection>, and examine its relation to other problems such as <Independent Set> and <Graph Coloring>. Consider for example an algorithm that:

Given a graph $G = (V, E)$, performs one of the following tasks in $O(m + n)$ (i.e., linear) time: (i) compute a $\Omega(1/\sqrt{n})$-approximation of <Maximum Independent Set> in G or (ii) find a triangle in G. The run-time is faster than that for any known method for each of these tasks.

The above result suggests the following broader research direction: if it is difficult to find (A) or (B) separately, can one find one of the two efficiently? This motivates the *dual pair* concept we introduce. We discuss and provide several instances of dual-pair approximation.

Weighted P-Median-Spanning Tree Problem

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Distributed sensor networks are comprised of two components where sensors gather information from the environment and routed to gateways which must share the data load seamlessly. We turn this structure into graphs where sensor-gateway relationship is organized through transportation edge weights and communication between gateways are linked to connection costs. Given that we have p gateways, each node has a sensor demand and a gateway deployment cost, we define a *weighted p-median-spanning tree problem* to find the optimal deployment of p gateways connected by a spanning tree and routing of each sensor to those gateways. We develop a baseline binary integer programming (BIP) model to tackle it. Since the model comprises computationally inefficient cycle elimination constraint, two alternative mixed integer linear programming (MILP) models and procedures based upon separation cuts are devised. We perform a computational analysis to test their performance.

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Critical subgraphs of Schrijver graphs for the fractional chromatic number

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Kneser graphs $\mathrm{KG}(n,k)$ are defined for every pair of positive integers n,k satisfying $n \geq 2k$. Their vertex set consists of the k-element subsets of $[n]$, two of which are adjacent if they are disjoint. The chromatic number of $\mathrm{KG}(n,k)$ is $n-2k+2$ and its fractional chromatic number is n/k. Generally, Kneser graphs are not vertex-critical for any of those parameters.

Schrijver graphs $\mathrm{SG}(n,k)$ are vertex-critical subgraphs of Kneser graphs for the chromatic number. They also share the value of their fractional chromatic number but Schrijver graphs are not critical for that either.

In this talk, we present an induced subgraph of every Schrijver graph $\mathrm{SG}(n,k)$ that is vertex-critical with respect to the fractional chromatic number. This subgraph is isomorphic with the circular complete graph $K_{n'/k'}$, where $\frac{n}{k} = \frac{n'}{k'}$ and $gcd(n',k') = 1$. We also characterize the critical edges within this subgraph.

The talk is based on the paper [1].

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Coupled task scheduling – minimizing the total completion time

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Coupled task problem on a single machine consists of scheduling a set of jobs, where each job has two tasks. These tasks have to be scheduled so that no tasks overlap and the two tasks of a job are scheduled with exactly their given delay time in between them.

We focus on the minimization of the total completion time of the jobs. Recently, [1] drew an almost full complexity picture for these problems. In this talk, we will present approximation algorithms for several NP-hard variants based on the results of [2].

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Cyclic scheduling with Timed Automata

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Timed Automata has been introduced by Alur and Dill in 1994[1] to model the behavior of real-time discrete-event-systems. Later, Behrmann et. al.[2] extended the mathematical model with a cost function to apply the available tools for optimization purposes. While MILP formulations and digraph based approaches dominate the field of production scheduling, several works have been published utilizing the Linearly Priced Timed Automata (LPTA) model for such purposes[3, 4, 5].

In this work, the applicability and scalability of the LPTA model for cyclic scheduling problems is examined, which poses a challenging modeling problem for traditional approaches.

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Sensitive χ^2 testing via sampling tripartite 3-uniform hypergraphs

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When assessing the independence of two categorical variables, the standard approach involves their bipartite interaction graphs transformed into contingency tables and analyzed using the χ^2 test. With three categorical variables, the interaction graph is a tripartite 3-uniform hypergraph, leading to a 3D contingency table. The χ^2 test remains applicable to assess the independence of any two variables of the trio through a 2D projection of the contingency table.

In this presentation, we introduce a more sensitive statistical test grounded in hypergraph theory. Specifically, we propose a hypergraph-based exact test that compares a χ^2 aggregation metric of the above interaction graph with a random sample of hypergraphs that share the same degree distribution.

In related research, the authors established the NP-hardness of sampling tripartite 3-uniform hypergraphs with prescribed degree distributions[1]. To address this challenge, we present a practical parallel tempering-based sampling method. We demonstrate, both on synthetic and real-world datasets, that the hypergraph-based exact χ^2 test consistently outperforms the conventional χ^2 test.

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New algorithms for generating Pareto-optimal points of multi-objective optimization problems

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In this talk, we present a new algorithm for generating Pareto-optimal points of multi-objective optimization problems. One of the cornerstones of the algorithm is the way in which the joint decreasing directions of the objective functions are determined. In our case, we use a linear programming auxiliary problem to determine (one of) the joint decreasing directions. The LP auxiliary problem can be solved efficiently in polynomial time. Variants of our new algorithm were also developed for different classes of multi-objective optimization problems like unconstrained, problems with sign restricted variables, and problems with linear constraints.

For each problem class, for the sequence of points produced by our algorithm during the solution of multi-objective optimization problems we prove that if we have an accumulation point then it is a substationary point, as well. In addition, if we assume that the objective functions are convex, then the substationary point is also a Pareto-optimal solution to the problem.

Conway’s Army Percolation

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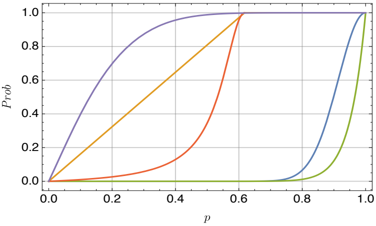
We study a probabilistic version of the celebrated Conway’s Army game. In this game the lower semiplane of an infinite chessboard is filled with checker pieces. A celebrated result due to Conway (1961) shows that one **cannot** move a checker piece in a finite number of steps to a position on row 5 (rows 1-4 are reachable).

In the version that we are interested in, inspired by percolation theory, we assign pieces to all cells of the lower semiplane independently by flipping a coin with success probability p (where p is a fixed constant between 0 and 1). We are interested in estimating $D_k(p)$, the probability that one can reach a fixed cell on line k, for $k = 1, 2, 3, 4$. We derive several results that provide lower and upper bounds on this probability. These results show that $0 < D_k(p) < 1$ for every $p \neq 0, 1$, and suggest the fact that the $D_k(p)$ is an analytical function of p. As a teaser for the talk, the results for $k = 4$ are summarized in the figure below. The green curve is a (direct) lower bound. The violet curve is a direct upper bound. The other curves are upper bounds based on tail bound inequalities: Markov’s inequality (yellow), Cantelli’s inequality (red), the Hoeffding inequality (blue). Despite visuals, the blue line is provably overtaken by the red line around $p = 0$ and by the violet line around $p = 1$.



An exact algorithm to find a maximum weight clique in a weighted undirected graph

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We introduce a new algorithm MaxCliqueWeight for identifying a maximum weight clique in a weighted graph, and its variant MaxCliqueDynWeight with dynamically varying bounds. This algorithm uses an efficient branch-and-bound approach with a new weighted graph coloring algorithm that efficiently determines upper weight bounds for a maximum weighted clique in a graph. We evaluate this algorithm on random weighted graphs with node counts up to 10,000 and on standard DIMACS benchmark graphs used in a variety of research areas. Our findings reveal a remarkable improvement in computational speed when compared to existing algorithms, particularly evident in the case of high-density random graphs and DIMACS graphs, where our newly developed algorithm outperforms existing alternatives by several orders of magnitude. The newly developed algorithm and its variant are freely available to the broader research community at http://insilab.org/maxcliqueweight, paving the way for transformative applications in various research areas, including drug discovery.

Transfer Learning: Boosting Machine Learning Efficiency and Efficacy

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In recent years, transfer learning has emerged as an innovation in machine learning, redefining the boundaries of data efficiency and model generalization. This technique, which involves the transfer of knowledge from one domain to another, has become crucial in addressing large, annotated datasets and in accelerating the training process for new tasks. The objective of this research is to provide a comprehensive examination of transfer learning methodologies, identify their potential in various applications, and highlight the challenges and frontiers of current research.

We begin by introducing the foundational concepts underlying transfer learning, differentiating between the paradigms of inductive, transductive, and unsupervised transfer learning. By dissecting the process of transferring and adapting features, models, or tasks, we provide insight into how prior knowledge can be used to enhance learning in a novel context.

We continue by critically analyzing the spectrum of transfer learning applications, ranging from natural language processing (NLP), where pre-trained models like BERT [1] have set new benchmarks, to computer vision, where models pre-trained on ImageNet [2] have demonstrated remarkable adaptability. We explore the efficacy of transfer learning in domains burdened with limited data availability, such as medical imaging and remote sensing.

To quantify the impact of transfer learning, we present a meta-analysis of benchmark datasets and tasks, providing empirical evi-

dence that supports the enhanced performance and reduced training times achieved by transfer learning strategies. The analysis reveals when transfer learning offers the most significant advantages, and under what circumstances it may be less effective.

However, transfer learning is not without challenges. We discuss the critical issues of negative transfer, where inappropriate knowledge transfer can degrade performance, the difficulty of identifying relevant source and target domains, and the computational burdens of fine-tuning large pre-trained models.

Finally, we take a look at the future of transfer learning research, emphasizing novel approaches such as few-shot learning, domain adaptation techniques, and cross-lingual transfer learning.

Keywords: Transfer Learning, Domain Adaptation, Pre-trained Models, Model Generalization, Machine Learning Efficiency.

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Disjunctive cuts and application to optimization with piecewise-linear functions

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Describing the convex hull of the union of a finite set of polyhedra is a fundamental problem of disjunctive programming. In the paper [1] a complete description is provided for so-called network-representable polyhedra. In the talk we present facet-separation algorithms, and also an application for computing with piecewise linear and convex $\mathbb{R}^3 \to \mathbb{R}$ functions.

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Generalized product-type probability approximations and bounds of higher order

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Approximating and bounding the probability of the realization of multiple events, which are not independent of each other are a problem with great interest in many fields.

In [2], [3], and [4] the authors introduced respectively the concepts of the cherry tree, multitree, and hypermultitree and the so-called m-regular hypergraphs to the calculation of bounds of the probabilities of the union of events.

Apart from these types of probability bounds, Block et. al. [1] proposed product type bounds. In this paper, we give more general product-type approximations of the probability of the union or intersection of events by using the characteristic random variables assigned to the events, and cherry tree graphs. Moreover, conditions under which these approximations are lower (upper) bounds will be also given.

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Compressing directed graphs with local symmetries

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We present a technique for the compression of directed graphs. As these graphs scale in size, they pose challenges in terms of storage and processing efficiency. We implemented two methods that compress directed graphs using local symmetries. These methods were subsequently combined with lossless compression algorithms and subjected to comparative analysis against state-of-the-art compression techniques tailored for directed graphs. Our evaluations were conducted on a dataset encompassing real-world graphs from various domains.

The results show the potential for improving the state-of-the-art methods by combining them with our approach, which is the basis for our future research.

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On strongly connected resolvable networks

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A Resolvable network (RN) is a data structure [1]. RNs are generalized digraphs. An RN contains subnetworks, a subnetwork contains nodes, and a node itself can be a subnetwork.

In an RN, subnetworks should not be distinct, they can overlap, but only distinct subnetworks can be connected by directed edges. The inner structure of subnetworks is unknown. Or in other words, we do not use the information on the inner structure of subnetworks to study RNs; but we use this information: how they are intersected by other subnetworks and connected to other subnetworks.

A reach consists of two connected subnetworks. A reach, say $A \to B$, can be represented by a clause, where nodes in A are represented by negative literals and nodes in B are represented by positive literals. A resolvable network may contain two special subnetworks: $Source$ and $Sink$. $Source$ has the property that there is no incoming edge to it. The $Sink$ has the property that there is no outgoing edge from it. The reach $Source \to B$ is represented by the clause, where nodes of B are represented by positive literals. The reach $A \to Sink$ is represented by the clause, where nodes of A are represented by negative literals. Any RN can be represented as a SAT problem. Furthermore, any SAT problem can be represented by an RN.

Since RNs are generalized digraphs, we can study the question: How to generalize notions of digraphs to the level of RNs? In this work, we generalize the notion of a strongly connected digraph. A digraph is strongly connected if and only if there is a path from any node to any other one [2]. It is not trivial to generalize this notion, since the notion of path is defined in the level of RNs. Therefore, we give a recursive list of RNs that are strongly connected. These examples have the following properties: A) Neither the *Source* nor the *Sink* is present. B) Their SAT representation has an equivalent SAT problem which contains only binary clauses, and each clause contains exactly one negative literal, and exactly one positive literal, and this SAT problem is represented by a strongly connected digraph. This second property ensures that the resulting RN will be a digraph, since each of its subnetworks are singleton sets, i.e., they contain only one node.

This definition is not very practical, because there is no practical algorithm that can generate an equivalent 2-SAT problem out of a general SAT problem. On the other hand, our recursive list of strongly connected RNs might help researchers to further analyze this problem.

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Exact and Heuristic Methods for the Oven Scheduling Problem

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The Oven Scheduling Problem is a new parallel batch scheduling problem that arises in the area of electronic component manufacturing. Jobs need to be scheduled to one of several ovens and may be processed simultaneously in one batch if they have compatible requirements. The scheduling of jobs must respect several constraints concerning eligibility and availability of ovens, release dates of jobs, setup times between batches as well as oven capacities. Running the ovens is highly energy-intensive and thus the main objective, besides finishing jobs on time, is to minimize the cumulative batch processing time across all ovens.

We propose to solve this NP-hard scheduling problem using constraint programming and integer linear programming techniques and present two different modelling approaches, one based on batch positions and another on representative jobs for batches. Additionally, we propose an approach based on simulated annealing to solve larger instances. An extensive experimental evaluation of our solution methods is performed on a diverse set of problem instances. We show that our methods can find feasible solutions for instances of realistic size, many of those being provably optimal or nearly optimal solutions.

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Optimal Cutting Arrangements in 1D

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Given a warehouse stocked with steel rods of potentially varied lengths, we face the challenge of fulfilling incoming orders for rod pieces. The inherent cost associated with cutting these rods necessitates minimizing the number of cuts. A primary strategy we adopt is identifying exact fits – collections of orders that perfectly match and utilize the entirety of a warehouse rod. This approach not only matches the order but also conserves resources by eliminating the need for additional cuts to discard the ”leftover rod segment.”

In this presentation, we will explore both the theoretical and practical aspects of the above problem.

In the theoretical section, we will explore the relationship between the feasible solution and the optimal solution. Furthermore, we will prove an equivalent formulation of the problem, which will help us find a reduction of this problem to optimal bin-packing. This latter problem is known to be NP-complete; thus, no polynomial-time solution for our problem is currently known.

For the practical section, we will introduce two approaches. The first employs dynamic programming combined with clique search. The second approach uses a 0-1 integer programming formulation, which turns out to be much more efficient than the first method. Our implementation of the second approach, utilizing the Gurobi Solver, will be demonstrated using simulated data. A potential extension of the 0-1 integer programming using hierarchical optimization will also be discussed.

Acknowledgments: We wish to extend our gratitude to the Slovenian–Hungarian applied mathematics joint project for introducing this problem to us.

Upper clique transversals in graphs: Complexity and algorithms

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A *clique transversal* in a graph is a set of vertices intersecting all maximal cliques. The problem of determining the minimum size of a clique transversal has received considerable attention in the literature. We initiate the study of the “upper” variant of this parameter, the *upper clique transversal number*, defined as the maximum size of a minimal clique transversal. We investigate this parameter from the algorithmic and complexity points of view, with a focus on various graph classes. We show that the corresponding decision problem is NP-complete in the classes of chordal graphs, chordal bipartite graphs, and line graphs of bipartite graphs, but solvable in linear time in the classes of split graphs and proper interval graphs.

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Digital Convexity based on Path Counting

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Digital geometry differs from Euclidean geometry as digital objects built up by pixels and it is not straightforward to define the digital analogs of various geometric concepts. For example, the Gauss digitization of a convex shape may not be connected [3]. Thus, there are various approaches to define digital convexity [1, 2].

The usual definition of convexity in the Euclidean plane is as follows: for any two points of the object all points of the shortest path between them belong to the object. However, in the digital scenario, in a grid, usually the shortest path is not unique. Thus, we may allow various digital analogue definitions. Obviously, we may define a (maximal) digital convexity, by requiring the object to contain the points (i.e., pixels, in this case) of each shortest path between any pairs of pixels of the object. On the other hand, a (minimal) digital convexity can be defined by requiring that the object contains the pixels of at least one shortest path between any pairs of pixels. Between this two extremal cases, we also show a digital convexity concept where the object must have at least the pixels of the half of the possible shortest paths between any two pixels. The shapes of these convex objects are characterized.

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MILP models of mixed Hamming packings: improved upper bounds

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We consider mixed Hamming packings, minimum codeword Hamming distances of mixed codes, using mixed integer programming. We introduce a reduction technique based on our idea of adopting the notion of contact graphs, motivated by continuous sphere packing problems. Our reduction technique helps in solving the respective mixed integer programs efficiently. Using the technique, we have improved various best known upper bounds of maximal cardinalities of Hamming packings with a given minimum distance for binary-ternary codes. Our approach can work for bigger problem instances, and is not restricted to binary-ternary codes. In spite of the limited number of variables in the models they are challenging for classical solvers. This suggests that their further study may yield benchmark problems for quantum computers that bear practical relevance.

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Optimization Algorithms in Precision Agriculture - Selected Use Cases

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The increase in world population makes it necessary to enhance the efficiency of food production, and agricultural tasks in general. Therefore, optimization problems arise in many segments of precision agriculture, and given their complexity, advanced heuristic algorithms are needed for their solution. Here, I will present two important tasks and their suggested solutions. The first task is the optimization of crop planting time, with the objectives of improving both effectiveness and efficiency of the production. The second task is the optimization of unmanned ground vehicle routing through blueberry fields, which must account for the characteristics of the field and the UGV. To solve both problems, we utilize heuristics based on adaptive large neighborhood search [1]. In the former, NSGA-II [2] is also used.

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An intermediate case of exponential multivalued forbidden matrix configuration

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The *forbidden number* $\mathrm{forb}(m, F)$, which denotes the maximum number of unique columns in an m-rowed $(0, 1)$-matrix with no submatrix that is a row and column permutation of F, has been widely studied in extremal set theory. Recently, this function was extended to r-matrices, whose entries lie in $\{0, 1, \ldots, r-1\}$. $\mathrm{forb}(m, r, F)$ is the maximum number of distinct columns of an r-matrix with no submatrix that is a row and column permutation of F. While $\mathrm{forb}(m, F)$ is polynomial in m, $\mathrm{forb}(m, r, F)$ is exponential for $r \geq 3$. Recently, $\mathrm{forb}(m, r, F)$ was studied for some small $(0, 1)$-matrices F, and exact values were determined in some cases. In this paper we study $\mathrm{forb}(m, r, M)$ for $M = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, which is the smallest matrix for which this forbidden number is unknown. Interestingly, it turns out that this problem is closely linked with the following optimisation problem. For each triangle in the complete graph K_m, pick one of its edges. Let m_e denote the number of times edge e is picked. For each $\alpha \in \mathbb{R}$, what is $H(m, \alpha) = \max \sum_{e \in E(K_m)} \alpha^{m_e}$? We establish a relationship between $\mathrm{forb}(m, r, M)$ and $H(m, (r-1)/(r-2))$, and in the case $r = 3$, prove lower and upper bounds for $H(m, 2)$ and use it to bound $\mathrm{forb}(m, 3, M)$ away from known general upper and lower bounds.

Abstract for the SWORDS 2023 presentation:
Cliqueful graphs as a means of calculating the maximum number of maximum cliques of simple graphs

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Simple graphs on n vertices may contain a lot of maximum cliques (largest cliques in the graph) and maximal cliques (non-extendable cliques in the graph). But how many can they potentially contain? In 1965, Moon and Moser [1] have already calculated the maximum number of **maximal cliques** of simple graphs on n vertices. We show, that the maximum number of **maximum cliques** of simple graphs on n vertices is also the same number [2]:

$$M_n = \begin{cases} 3^{\lfloor n/3 \rfloor} \text{ if } n \bmod 3 = 0 \\ 4 \cdot 3^{(\lfloor n/3 \rfloor - 1)} \text{ if } n \bmod 3 = 1 \\ 2 \cdot 3^{\lfloor n/3 \rfloor} \text{ if } n \bmod 3 = 2 \end{cases}$$

For this, we introduce the family of so-called ”cliqueful” graphs: graphs that can be fully determined by their set of maximum cliques. Then we prove, that this set definitely contains graphs with the maximum number of maximum cliques on n vertices $\mathrm{MG}(n)$. We will do this by further reducing the set that contains $\mathrm{MG}(n)$, to so-called ”saturated cliqueful” graphs, and ”composite saturated cliqueful” graphs. It will turn out that $\mathrm{MG}(n)$, if $n \geq 15$, is *almost* the same as composite saturated cliqueful graphs with prime components of size ≤ 4.

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CliSAT, a new exact algorithm for the maximum clique problem

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The maximum clique problem (MCP) is a fundamental deeply studied $\mathcal{NP}$-hard problem in graph theory that finds numerous applications spanning different fields, such as, robotics, biochemistry, computer vision and many others. In the last two decades, the performance of exact algorithms have increased by orders of magnitude and I have had the fortune of participating in the exciting *horse race* for the most efficient algorithm. Along this path, many interesting upper bounds and branching techniques have been described. This talk summarizes the main components of the recent combinatorial branch-and-bound algorithm CliSAT [1] for the MCP.

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Shannon capacity and Hedetniemi-type equalities

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The categorical product of two graphs F and G, denoted $F \times G$, satisfies for all homomorphism monotone (increasing) graph parameters φ that $\varphi(F \times G) \leq \min\{\varphi(F), \varphi(G)\}$. Although it is famously not true for the chromatic number by the recent disproof of Hedetniemi's conjecture, some parameters give equality in this inequality. Examples of such parameters are the fractional chromatic number and the complementary Lovász number by results in [3] and [1], respectively. Both of these parameters are elements of what Zuiddam [4] calls the asymptotic spectrum of graphs. In the talk, that is based on [2], we elaborate on the consequences of the main result in [4] concerning the possibility of Shannon's graph capacity, the zero-error capacity of a noisy channel with distinguishability graph G, satisfying this Hedetniemi-type equality.

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Additional pruning and backtracking rules in the Carradhan-Pardalos algorithm applied to packing by cubical clusters

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The problem of packing optimally a large cube by translated copies of a tripod can be reduced to a clique search problem. As a first step one constructs a suitable compatibility graph G. Then one feeds this graph G into a clique solver. In our case, we will use a version of the Caraghan–Pardalos algorithm. The procedure works with two subsets of the compatibility graph G. Namely, the clique under construction C and the set of prospective nodes P. One picks a vertex v of P and extends by adding v to C to get a larger clique and reduces P to the common neighbors of the elements of C. If P is empty then the search backtracks. One may anticipate backtracking before exhaustingly testing each element of P. We refer to this action as pruning of the search tree. The main result of this work is following. We define a directed graph D whose nodes are the vertices of the compatibility graph G. We show that if T is an optimal clique in G, then there is a clique T' such that the node set of T' induces a connected component in D. We can exploit D to speed up the Carraghan–Pardalos algorithm. If a vertex v in P is not an initial point of a directed edge of D whose terminal point is in $C \cup P$, then v can be deleted from P. If a vertex v in C is not an initial point of a directed edge of D with an end point in $C \cup P$, then we may backtrack. We carry out numerical experiments to test the practical utility of the suggested pruning and backtracking rules.

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Privacy preserving fall detection using homomorphic encryption

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Accidental falls pose a risk to the health and independence of older adults. According to the World Health Organization's report titled "Fall Prevention in Older Age" [7], around 30 % of individuals aged 65 experience falls annually, and this risk rises for those above 70 years old. Despite various factors contributing to fall prevention (World, 2008), falls can sometimes result from underlying health issues, making them difficult to prevent entirely. Hence, timely detection of fall incidents is crucial to averting severe consequences stemming from fall-related injuries and other hazardous situations.

Tošić et al. [5] present a non-intrusive fall detection solution based on the smart floor, the same setting can be extended into an indoor location system and authors also argue a vast spectrum of possible applications. Hrovatin et al. [4] present local computation obscured by onion routing so only results of the computation leave the nodes ensuring the data privacy by never moving the data from the nodes. An additional possible way to deal with the privacy problems (preserving privacy) is the to use machine learning approaches/algorithms on specially encrypted data using homomorphic encryption [6].

The machine learning algorithm used in this experiment was Random forest [2], more precisely its evolved Python implementation in Catboost [3] library due to expected low discrepancy of the algorithm on encrypted data. The algorithm is supposed to perform within marginal differences on the encrypted data as concluded in a recent study by Matias et al. [1].

We report acceptable results on our test-setting both from the accuracy (performance) view point and fro the time complexity point of view.

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Analysis of quantities related to COVID-19 pandemic

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The presentation will review the course of the COVID-19 pandemic through numerical calculations in 17 countries covering the continents. Countries show surprisingly high degree of similarity over time in total cases, the amount of vaccines used, the number of vaccinated people and the number of fully vaccinated individuals. This provides an opportunity to develop inventory models that can be used in different countries. The first model seeks to minimize warehousing costs, while the second model guarantees security of supply at a given probability level with minimal storage costs.

Keywords: COVID-19 epidemic, saturation process, Cachy distribution, minimum warehousing costs, Hungarian inventory management model.

From Szeged number to closeness of networks

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Szeged number [2] is a topological index of molecular graphs studied in chemical graph theory. In fact, it is perhaps a more natural generalization of the original Wiener number defined on trees. The usual generalization, called the Wiener number of a graph, is just the sum of all distances. Needless to say, the Wiener number and its generalizations are of interest when considering the communication networks. Hence, the Hosoya polynomial [3], whose derivative at $x = 1$ is the Wiener number, may be of general interest. We know that Hosoya polynomial can be computed in linear time on double weighted cactus graphs [4], likely also on some larger classes of networks. An example illustrating the use of Hosoya polynomial is its close relation to the recently defined measure of network centrality, namely the network closeness [1]. Based on the concept of closeness, decay stable graphs were defined in [5]. We prove one and discuss another conjecture of [5].

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